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USE OF PARALLEL POLARIZED RADIATION IN DETERMINATIONS OF OPTICAL CONSTANTS AND THICKNESS OF FILMS

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INTRODUCTION

A KNOWLEDGE of the optical constants of dielectric coatings and films is relevant to the calculation of radiation properties such as reflectivity and absorptivity which, in turn, are employed to determine radiative heat transfer. Several methods are developed in [1] for deducing optical constants and film thickness from monochromatic, specular reflectivity measurements at varying angle of incidence. The methods described therein were formulated for perpendicular polarized radiation. The purpose of the present note is to extend the formulation to accommodate parallel polarized radiation.

The physical situation under study is pictured schematically in the inset of Fig. 1. Consideration is given both to transparent films and to slightly absorbing films. Although the former correspond to the conventional analytical model for thin dielectric films, the experiments of [1] indicate that the latter may provide a closer representation to reality. To facilitate a concise presentation, liberal use will be made of the findings of [1].

TRANSPARENT FILMS

Consider first a transparent film ($\hat{n}_2 = n_2$) on an absorbing substrate [$\hat{n}_3 = n_3(1 + i\kappa_3)$]. For monochromatic, parallel polarized radiation incident under an angle θ_1, the Fresnel reflection coefficients at the interfaces 1–2 and 2–3, r_{12} and ρ_{23}, respectively, are

$$r_{12} = \frac{n_2 \cos\theta_1 - n_1 \cos\theta_2}{n_2 \cos\theta_1 + n_1 \cos\theta_2},$$

$$\rho_{23}\, e^{i\phi_{23}} = \frac{\hat{n}_3 \cos\theta_2 - n_2 \cos\theta_3}{\hat{n}_3 \cos\theta_2 + n_2 \cos\theta_3} \tag{1}$$

in which ϕ_{23} is the phase shift at the interface 2–3. The angles θ_1, θ_2 and θ_3 are related by Snell's law, according to which $n_1 \sin\theta_1 = n_2 \sin\theta_2 = \hat{n}_3 \sin\theta_3$.

At the Brewster angle defined by $\theta_1 = \theta_1^* = \tan^{-1}(n_2/n_1)$, it can be shown that $r_{12} = 0$. Furthermore, for situations characterized by $n_1 \cong 1$ and $n_2 > 1$, it follows that $r_{12} > 0$ for $\theta_1 < \theta_1^*$ and $r_{12} < 0$ for $\theta_1 > \theta_1^*$. This behaviour of r_{12} introduces interesting differences in the present development relative to that for perpendicular polarized radiation.

In terms of the foregoing quantities, the monochromatic specular reflectivity is expressible as

$$R = \frac{r_{12}^2 + \rho_{23}^2 + 2r_{12}\rho_{23}\cos(\phi_{23} + 2\beta)}{1 + r_{12}^2\rho_{23}^2 + 2r_{12}\rho_{23}\cos(\phi_{23} + 2\beta)},$$

$$\beta = \left(\frac{2\pi}{\lambda_0}\right) n_2 h \cos\theta_2 \tag{2}$$

where λ_0 is the wavelength (in vacuum) of the incident radiation. It is seen by examination of equation (2) that for fixed values of the optical constants and for a given angle of incidence θ_1, the reflectivity R is a periodic function of the film thickness† h. The maxima and minima attained by the periodically varying function $R = R(h)$ are independent of h and are found from the condition $\partial R/\partial h = 0$, which gives

$$\phi_{23} + 2\beta = m\pi, \quad m = 0, \pm 1, \pm 2, \ldots \qquad (3)$$

so that, from equation (2)

$$R_{\min,\max} = [(\rho_{23} + (-1)^m r_{12})/(1 + (-1)^m r_{12}\rho_{23})]^2. \qquad (4)$$

For $\theta_1 < \theta_1^*$ where $r_{12} > 0$, $R_{\min}$ corresponds to odd values of m while $R_{\max}$ corresponds to even values of m. On the other hand, for $\theta_1 > \theta_1^*$, odd and even m correspond respectively to $R_{\max}$ and $R_{\min}$.

wavelength. The reflectivity curves are bounded by the envelope curves. When a reflectivity curve is tangent to an envelope curve, it attains $R_{\max}$ or $R_{\min}$. Also, at the Brewster angle, the envelope curves and the reflectivity curves are coincident, regardless of the film thickness h.

The just discussed characteristics may now be employed to formulate a method for deducing the refractive index n_2 of the film from measured reflectivity data, the film thickness being unknown. Suppose that a reflectivity curve has been determined experimentally and plotted in a R, θ_1 diagram, for example, one of the reflectivity curves of Fig. 1. Then, for the given wavelength λ_0 and n_1, and for known properties of the substrate (i.e. n_3, κ_3), a succession of envelope curves can be evaluated from equation (4) for parametric values of n_2. That value of n_2 which gives rise to a condition of tangency between the reflectivity curve

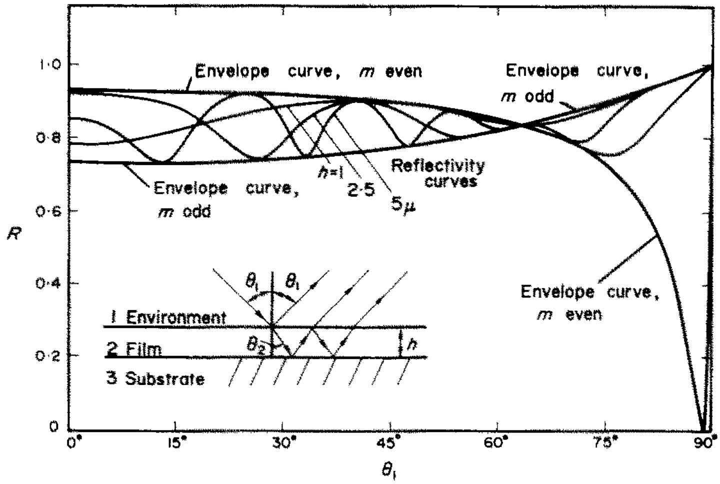

FIG. 1. Illustrative envelope curves and reflectivity curves.

For given optical constants n_1, n_2, n_3, κ_3 and a given wavelength λ_0, equation (4) can be employed to generate a pair of envelope curves in a diagram of R vs. θ_1. Such envelope curves are illustrated in Fig. 1, where $n_1 = 1$, $n_2 = 2$, $n_3 = 1{\cdot}21$, $\kappa_3 = 6{\cdot}15$, and $\lambda_0 = 0{\cdot}63\ \mu$ (the value $n_2 = 2$ is representative of a number of optical materials, whereas the n_3, κ_3 are for an aluminum substrate). The curves are seen to intersect at the Brewster angle θ_1^*, which, for the case under consideration, is approximately 63·5°.

In addition to the envelope curves, Fig. 1 contains three other curves, each of which depicts the variation of reflectivity with incidence angle for a film of thickness h and for the aforementioned values of the optical constants and

† Except at $\theta_1 = \theta_1^*$, where R is independent of h.

and the envelope curves is the refractive index of the film. An internal verification of the just-outlined procedure is afforded by the requirement that the reflectivity curve must pass through the point of intersection of the envelope curves.

It may also be observed from Fig. 1 that at angles smaller than the Brewster angle, the tangencies between the reflectivity curve and the envelope curves are essentially coincident with the minima and maxima of the former (defined by $\partial R/\partial\theta_1 = 0$). The sharper the minimum (or maximum), the more closely does it coincide with a point of tangency. If a point of tangency and a minimum of the reflectivity curve are assumed to be coincident, then n_2 can be calculated by an alternate procedure to that described above. The value of R and the angle θ_1 at a minimum point of the measured reflectivity curve are noted. With this information as input, and for given values of n_1, n_3, κ_3 and λ_0, the only unknown appearing in equation (4) is n_2, so that its value can be determined.

Once n_2 is known, then the film thickness h is determined by employing equation (3), which, after substitution of β from equation (2) and use of Snell's law, becomes

$$h = [\lambda_0/4\pi\sqrt{(n_2^2 - n_1^2 \sin^2\theta_1)}](m\pi - \phi_{23}) \qquad (5)$$

where ϕ_{23} is an algebraic function of n_1, n_2, n_3, κ_3 and θ_1, all of which are known. The procedures for determining h from equation (5) are the same as those used in connection with equation (10) of [1].

SLIGHTLY ABSORBING FILMS

For the case of an absorbing film $[\hat{n}_2 = n_2(1 + i\kappa_2)]$ on an absorbing substrate $[\hat{n}_3 = n_3(1 + i\kappa_3)]$, the Fresnel reflection coefficients ρ_{12} and ρ_{23} for monochromatic, parallel polarized radiation are given by

$$\rho_{12}\, e^{i\phi_{12}} = \frac{\hat{n}_2 \cos\theta_1 - n_1 \cos\theta_2}{\hat{n}_2 \cos\theta_1 + n_1 \cos\theta_2},$$

$$\rho_{23}\, e^{i\phi_{23}} = \frac{\hat{n}_3 \cos\theta_2 - \hat{n}_2 \cos\theta_3}{\hat{n}_3 \cos\theta_2 + \hat{n}_2 \cos\theta_3} \qquad (6)$$

where ϕ_{12} and ϕ_{23} are corresponding phase shifts. In turn, the specular reflectivity R is expressible as

$$R = \frac{\rho_{12}^2 e^{2v_2\eta} + \rho_{23}^2 e^{-2v_2\eta} + 2\rho_{12}\rho_{23}\cos(\phi_{23} - \phi_{12} + 2u_2\eta)}{e^{2v_2\eta} + \rho_{12}^2\rho_{23}^2 e^{-2v_2\eta} + 2\rho_{12}\rho_{23}\cos(\phi_{23} + \phi_{12} + 2u_2\eta)} \qquad (7)$$

in which $\eta = (2\pi/\lambda_0)h$, and u_2 and v_2 are algebraic functions of n_1, n_2, κ_2 and θ_1 expressed by equation (23) of [1].

Consideration is now given to slightly absorbing films, that is, $\kappa_2 \ll 1$. To assist in visualizing the effect of small κ_2 on the reflectivity distribution, equation (7) has been numerically evaluated and the corresponding results are plotted in Figs. 2 and 3. Each figure shows the variation of R as a function of the incidence angle θ_1 for parametric values of κ_2. Figure 2 is for $h = 2{\cdot}5\ \mu$, while Fig. 3 is for $h = 5\ \mu$. In both figures, $n_1 = 1$, $n_2 = 2$, $n_3 = 1{\cdot}21$, $\kappa_3 = 6{\cdot}15$ and $\lambda_0 = 0{\cdot}63\ \mu$.

The figures show a rather remarkable result, namely, that the magnitude of the reflectivity is strongly affected† by small κ_2, but the angular positions of the maxima and minima are insensitive to κ_2. In particular, the angles at which the maxima and minima occur for small values of κ_2 are essentially the same as those for $\kappa_2 = 0$.

The foregoing observation forms the basis for a method for deducing n_2, κ_2 and h from a measured distribution of monochromatic, specular reflectivity as a function of incidence angle. Among the minima and maxima of the reflectivity curve, two are selected and their angular positions are identified as $\theta_{1(1)}$ and $\theta_{1(2)}$. It is then assumed that these $\theta_{1(1)}$ and $\theta_{1(2)}$ also mark the positions of the corresponding extremums in the R, θ_1 distribution for an otherwise identical

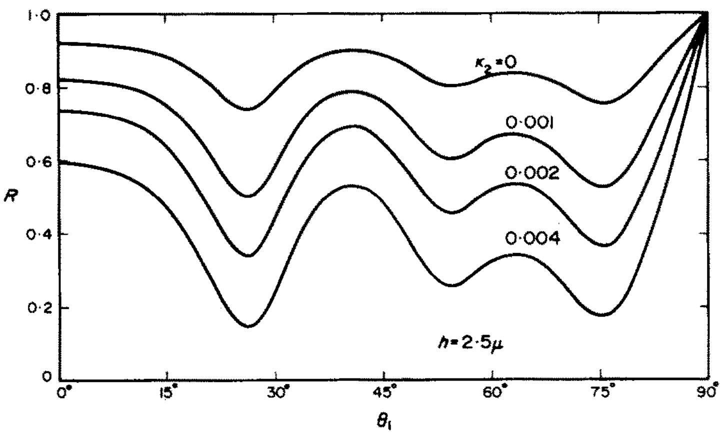

FIG. 2. Effect of κ_2 on the reflectivity, $h = 2{\cdot}5\ \mu$.

† Owing to this effect of small κ_2, reflectivity curves corresponding to films of different thickness will not intersect at the Brewster angle. Consequently, the method of Abeles [2] for determining n_2 for transparent films does not apply for slightly absorbing films.

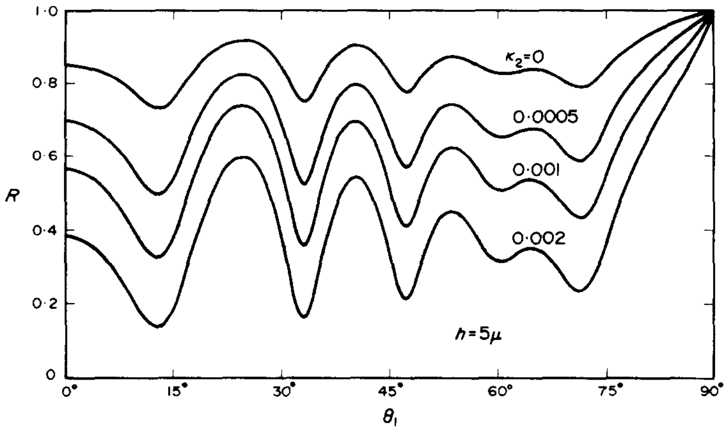

FIG. 3. Effect of κ_2 on the reflectivity, $h = 5\ \mu$.

film-substrate system but with $\kappa_2 = 0$. Furthermore, it is additionally assumed that the $\theta_{1(1)}$ and $\theta_{1(2)}$ can be associated with the angular positions of tangencies between the $\kappa_2 = 0$ reflectivity curve and the envelope curves.

With these postulates, the calculation procedure outlined in [1] for perpendicular polarized radiation can now be employed for parallel polarized radiation. In the execution of the method, equation (10) of the reference is replaced by equation (5) with ϕ_{23} from equation (1). The assignment of even or odd values to m is based on the rules stated in the text following equation (4).

Experiments described in [1] have established the practical utility of these methods for determining the optical constants and thickness of thin films.

ACKNOWLEDGEMENT

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ON A SIMPLE CORRELATION FOR PRANDTL NUMBER EFFECT ON FORCED CONVECTIVE HEAT TRANSFER WITH SECONDARY FLOW

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1. INTRODUCTION

THE PRANDTL number effect on fully developed laminar forced convection heat transfer with secondary flow was studied recently for uniformly heated horizontal tubes [1]